A New Figure of Merit of LC Oscillators Considering Frequency Tuning Range

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***Abstract*— In this paper, a new Figure of Merit (FoM) of LC oscillators considering frequency tuning range is proposed. The oscillators which have wide frequency tuning range are demanded due to the recent wireless technology. FoM is often utilized to evaluate oscillators. However, the conventional FoM does not consider the frequency tuning range, which degrades the phase noise drastically. The proposed $\mathrm{FoM_L}$ can consider the degradation caused by wide frequency tuning range, which is derived from degradation in quality factor of inductor. The simulation results are presented with the analytical explanation, and these FoMs are compared using the results in reference papers.**

I. Introduction

Recently, focus on the performance of VCOs is increasing since they play a critical role in setting the performance of many building circuit blocks like RF front-ends and clock generators. As a benchmark to compare VCOs performance, FoM is widely used in literature. However, there is no FoM that takes into account degradation in Frequency Tuning Range (FTR) on physical basis and relates it to circuit parameters.
In this work, a new FoM is proposed, which takes into account the VCO FTR and relates it to circuit parameters. This paper goes as follows, section II describes how to derive the new FoM, named "$\mathrm{FoM_L}$". Section III shows a comparison between "$\mathrm{FoM_L}$" and the conventional FoM that includes the FTR, named "$\mathrm{FoM_T}$". Section IV shows the result of the comparison using several VCO papers.

II. New FoM Calculation

In this section, a new FoM is defined by including the deterioration of quality factor as the tuning range is widened into the conventional FoM. Initially, close look at the conventional FoM is given followed by the introduction of the new FoM in the second part.

A. FoM Considering Quality factor of the resonator

The conventional FoM for oscillators is given as follows [1]:

$$\mathrm{FoM} = \mathcal{L}(f_{\mathrm{offset}}) - 20\log\left(\frac{f_0}{f_{\mathrm{offset}}}\right) + 10\log\left(\frac{P_{\mathrm{DC}}}{1\mathrm{mW}}\right) \tag{1}$$

where $\mathcal{L}$ is the phase noise, f_0 is the oscillation frequency, f_{offset} is the offset frequency from the carrier, and P_{DC} is the power consumption. Moreover, the phase noise is defined as the ratio of noise power to signal power in a 1Hz bandwidth, which is given for a LC oscillator by [2]:

$$\mathcal{L}(f_{\mathrm{offset}}) = 10\log\left[\frac{2F\mathrm{k}T}{P_{\mathrm{sig}}}\frac{f_0^2}{4Q^2 f_{\mathrm{offset}}^2}\right] \tag{2}$$

where k is Boltzmann constant, T is the absolute temperature, P_{sig} is the output power, F is device excess noise factor and Q is the quality factor of the resonator. The following expression is obtained by substituting Eq. (2) in Eq. (1):

$$\mathrm{FoM} = 10\log\left[\frac{F\mathrm{k}T}{2\mathrm{mW}}\frac{P_{\mathrm{DC}}}{P_{\mathrm{sig}}Q^2}\right] \tag{3}$$

It can be seen from Eq. (3) that the FoM is mainly determined by P_{DC}, P_{sig} and Q. Since $P_{\mathrm{DC}}/P_{\mathrm{sig}}$ is usually fixed and determined by a circuit topology, we only take a closer look at the quality factor which is given by:

$$Q = \frac{Q_{\mathrm{L}}Q_{\mathrm{C}}}{Q_{\mathrm{L}} + Q_{\mathrm{C}}} \tag{4}$$

where Q_{L} is the inductor quality factor, and Q_{C} is the capacitor one. Generally, quality factor of capacitors is much higher than that of inductors, i.e. $Q_{\mathrm{C}} \gg Q_{\mathrm{L}}$. Thus, Eq. (4) can be simplified as :

$$Q \approx Q_{\mathrm{L}} \tag{5}$$

It can be shown that the quality factor of the inductor is given by the parasitic resistance $\mathrm{R_L}$, inductance L and frequency f as:

$$Q_{\mathrm{L}} = \frac{2\pi f\mathrm{L}}{\mathrm{R_L}} \tag{6}$$

Therefore, inductor quality factor changes with frequency. For a VCO, the mean value of the quality factor can be found from Eq. (6) by considering the average between the maximum oscillation frequency $f_{\max}$ and the minimum $f_{\min}$ as follows:

$$Q_{\mathrm{avg}} = \frac{2\pi\mathrm{L}}{\mathrm{R_L}}\frac{f_{\max} + f_{\min}}{2} \tag{7}$$

Moreover, FTR can be expressed as $(f_{\max}+f_{\min})/f_{\mathrm{center}}$. Therefore, when FTR widens, it is necessary to decrease $f_{\min}$ and thus the performance of the VCO deteriorates since the average quality factor (Q_{avg}) is reduced. Thus, when considering FTR, it is indispensible to think about the deterioration of the inductor quality factor. Currently, FoM_{T} is used to include FTR into the conventional FoM given by [3]:

$$\text{FoM}_{\text{T}} = \text{FoM}_{\text{peak}} - 20\log\left(\frac{FTR}{0.1}\right) \tag{8}$$

where FoM_{peak} is calculated from [1] without taking the aforementioned effect into account. Moreover, this FoM has no physical meaning that relates directly to circuit parameters. Therefore, it is necessary to define a new FoM in consideration of quality factor deterioration with FTR.

B. New FoM Calculation

It can be seen from Eq. (6) that wider FTR results in lower average Q, i.e. lower quality factor at the center frequency. So it is necessary to estimate the reduction of the quality factor as f_{min} and f_{center} are made lower.
Eq. (6) can be rewritten using constant k_{L} as:

$$Q(f) \quad = \quad \frac{2\pi \text{L}}{\text{R}_{\text{L}}} \cdot f \equiv \text{k}_{\text{L}} \cdot f \tag{9}$$

Using $f_{\text{center}} = (f_{\text{max}} + f_{\text{min}})/2$, the formula is derived as follows:

$$\begin{aligned} f_{\text{max}} &= f_{\text{center}} + \frac{f_{\text{max}} - f_{\text{min}}}{2} && (10)\\ &= f_{\text{center}}\left(1 + \frac{FTR}{2}\right) && (11) \end{aligned}$$

Eq. (9) can be used to relate $Q(f_{\text{center}})$ and $Q(f_{\text{max}})$ as follows:

$$\begin{aligned} Q(f_{\text{center}}) &= \text{k}_{\text{L}} \cdot f_{\text{center}} && (12)\\ &= \frac{Q(f_{\text{max}})}{f_{\text{max}}} \cdot f_{\text{center}} && (13) \end{aligned}$$

by substituting Eq. (11) in Eq. (13) we get:

$$Q(f_{\text{center}}) \quad = \quad \frac{Q(f_{\text{max}})}{1 + \frac{FTR}{2}} \quad (< Q(f_{\text{max}})) \tag{14}$$

Eq. (14) shows that the quality factor at the center frequency drops when FTR is made wider.
Finally, the new FoM, named "FoM_{L}", is defined by taking the degredation in the quality factor as follows:

$$\begin{aligned} \text{FoM}_{\text{L}} &= 10\log\left[\frac{FkT}{2\text{mW}} \frac{P_{\text{DC}}}{P_{\text{sig}}\left\{Q(f_{\text{center}}) \cdot \left(1 + \frac{FTR}{2}\right)\right\}^2}\right] \\ \therefore \text{FoM}_{\text{L}} &= \text{FoM}(f_{\text{center}}) - 20\log\left(1 + \frac{FTR}{2}\right) && (15) \end{aligned}$$

III. Simulation

In section II, FoM_{L} was defined in consideration of the change in Q due to FTR. FoM and FoM_{T} were also compared and through verification it was shown that the difference actually goes out. In this section, FoM is examined both by first doing theoretical analysis and then by showing simulation results afterwards about each difference. The circuit that is used for comparing both FoMs is given in Fig.1. It is a typical NMOS cross coupling type VCO as shown in the figure.

A. Numerical Comparison

Since the average Q degrades as FTR widens, a good FoM should remain relatively constant as FTR widens. The quality factor of the inductor is assumed to be 10, and the absolute temperature T is assumed to be 300K. Moreover, channel length modulation is neglected and device excess noise factor F is calculated for the NMOS cross couple type as $(1 + 2/3)$ [2]. The assumption in Eq. (5) is also adopted here for the resonator quality factor. Therefore, FoM_{peak} is equal to FoM_{L} since FoM_{L} is calculated such that it does not depend on the FTR. Comparison results between FoM_{L}, FoM_{T} and FoM at the center frequency $\text{FoM}_{\text{center}}$ are given in Fig.2. It can be seen from the figure that the wider the FTR, the worse is $\text{FoM}_{\text{center}}$. However, FoM_{L} remains constant as FTR widens while FoM_{T} improves.

B. Simulation Result

Simulation is used to confirm the results obtained in Fig.2 to see whether it correctly predicts the behavior of oscillator as FTR widens. The circuit in Fig.1 is simulated using a 0.18μm CMOS process for two inductors that has a maximum Q at 10GHz and 2.7GHz. Quality factor of the inductor is calculated from the following expression [4]:

$$Q_{\text{L}} = \frac{\text{Im}[Z_{\text{L}}]}{\text{Re}[Z_{\text{L}}]} \tag{16}$$

where Z_{L} is the impedance of the inductor. Quality factor curves for both inductors are given in Fig.3 and summarized in Table I:

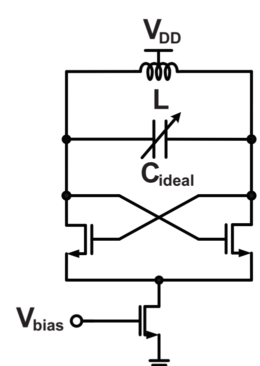

Fig. 1. VCO circuit in simulation

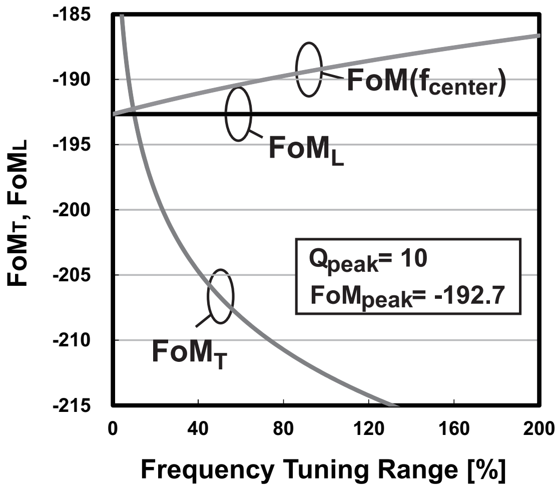

Fig. 2. Theoretical curve of each FoMs

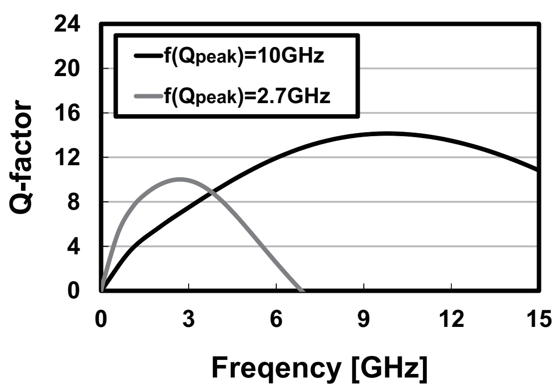

Fig. 3. Quality factor of inductor

FTR is changed by the capacitor value where ideal capacitors were used in simulation. The results are given in Fig.4 which confirms the deterioration in FoM as FTR widens. From the figure also it can be seen that $\mathrm{FoM_L}$ remains relatively constant as FTR widens while $\mathrm{FoM_T}$ improves. Therefore, $\mathrm{FoM_L}$ gives a better indication for the performance of the VCO.

IV. Paper Comparison

A group of published VCO papers were used to confirm the results of the theory and simulations. Results were calculated based on Eq. (8) and Eq. (15) and summarized in Fig.5 ([5]–[17]). From the figure it can be seen that $\mathrm{FoM_T}$ improves as FTR widens. On the other hand, $\mathrm{FoM_L}$ takes the effect of the deterioration in the average Q and remains relatively constant.

V. Conclusion

In this work, a new FoM for voltage-controlled oscillators (VCOs) is proposed. It takes into account the degradation in average Q of the inductor as FTR widens. Simulation results and paper comparison were used to confirm the validity of the proposed FoM where it was found that it gives a better figure of merit for comparing the performance of VCO.

TABLE I
Spec of inductor

	L_1	L_2
Inductance [nH]	1.77	5.23
Frequency of Q_{peak} [GHz]	10.1	2.7
Q_{peak}	14	11

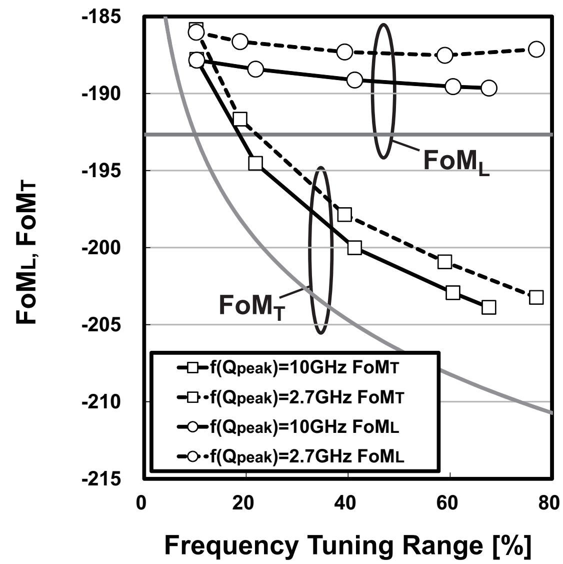

Fig. 4. Simulation Result

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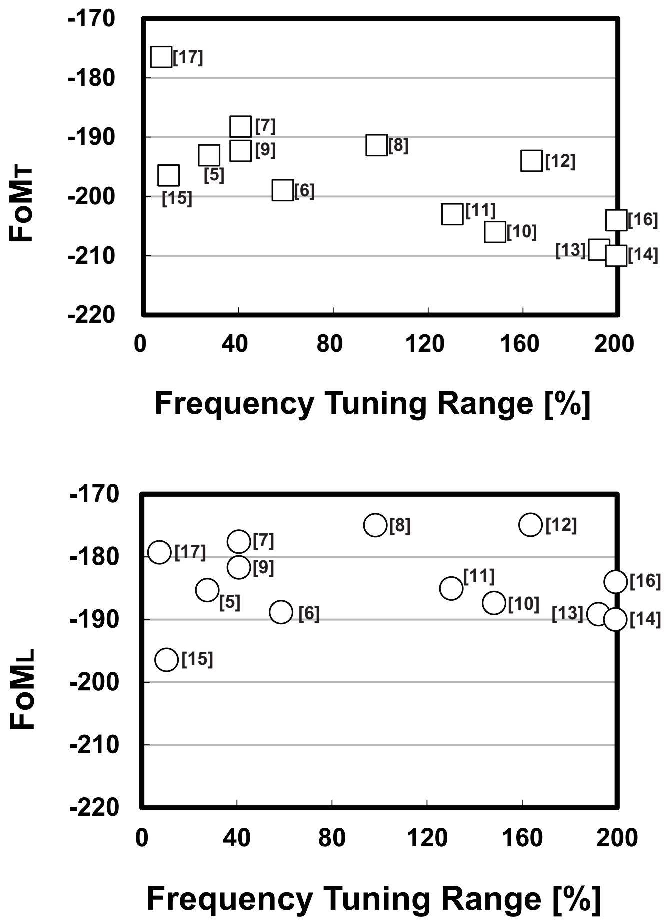

Fig. 5. Comparison $\mathrm{FoM_L}$ with $\mathrm{FoM_T}$

■ **Definition**

$$\mathrm{FoM_L} = \mathrm{FoM}(f_{\mathrm{center}}) - 20\log\left(1 + \frac{FTR}{2}\right)$$

$$f_{\mathrm{center}} = \frac{f_{\mathrm{max}} + f_{\mathrm{min}}}{2}$$

$$FTR = \frac{f_{\mathrm{max}} - f_{\mathrm{min}}}{f_{\mathrm{center}}}$$

$$\mathrm{FoM}(f_{\mathrm{center}}) = \mathcal{L}(f_0 = f_{\mathrm{center}}) - 20\log\left(\frac{f_{\mathrm{center}}}{f_{\mathrm{offset}}}\right) + 10\log\left(\frac{P_{\mathrm{DC}}}{1\mathrm{mW}}\right)$$

f_{max}: the highest oscillation frequency
f_{min}: the lowest oscillation frequency
f_{center}: the center frequency in the tuning range
FTR: frequency tuning range
f_{offset}: the offset frequency
$\mathcal{L}(f_{\mathrm{offset}})$: phase noise at the offset frequency foffset
P_{DC}: power consumption

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